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A New Tactic for Finding Irrelevant Constraints in Linear Programming Problems

DR.P.SUMATHI DR.P.BALACHANDER MRS.A.KARPAGAM MR.P.PUGAZHENTHI

Abstract—Redundancy in mathematical programming is a common occurrence, generally brought about by the lack of complete knowledge about the system of constraints and the desire on the part of the problem formulator not to omit essential elements of the formulation. .Something is redundant if it can be omitted without consequences for the system concerned. This paper presents a new method for selecting a constraint in linear programming problems to identify the irrelevant constraints. The computational results are presented and analyzed for small scale problems in this paper.

Index Terms— Linear programming, irrelevant constraints, redundant constraints, binding constraints, presolving, restrictive constraint,

1 INTRODUCTION

In solving an LPP, we tend to include all possible constraints thatwill increase the number of iterations and computational work. It is well known that for most of the large scale LP problems, only a relatively small percentage of constraints are binding at the optimal solutions. The purpose of this paper is to propose a new method to find the irrelevant constraints.

Redundancy may even have some favorable effects. Redundancy might also yield some theoretically interesting properties. It is our conviction, however, that in the general mathematical programming problem, the unfavorable effects of redundancy far outnumber the favorable ones.

In solving an LPP, it is acknowledged that redundancies do exist in most of the practical LPPs. The importance of detecting and removing redundancy in a set of linear constraints is the avoidance of all the calculations associated with those constraints when solving an associated LPP.

Many Researchers [1-16] have proposed different approach to identify the redundancies in linear programming problems.

The general linear programming model with bounded variables can be stated as

$$\text{LP: } \max Z = CX$$
$$\text{Subject to } AX \le b, \qquad (1)$$
$$0 \le X \le U$$

Where X is an n x 1 vector of variables. A is an m x n matrix $[a_{ij}]$ with 1 x n row vectors A_i, i = 1,2,3,...,m, b an m x 1 vector, C an 1 x n vector and 0 an n x 1 vector of zeros . U is an n x 1 vector.

In 1989,Caron et. al [5] proposed a theorem to identify the redundant constraints, which states that the k^{th} constraint, $A_kX \le b_k$ is redundant if and only if the problem LP_k has an optimal solution X^* with $A_kX^* \le b_k$,

[1]Research Scholar, Department of Mathematics, Easwari Engineering College, Ramapuram,Chennai – 89.
[2]Associate Professor, Head of Department, Valliammai Engineering College, Kattankulathur, Chennai.
[3]Assistant Professor, Department of Mathematics, Valliammai Engineering College, Kattankulathur, Chennai.
[4]Research Scholar, University of Madras, Chennai-5.

E-mail: psumathi16@rediffmail.com, balap27@gmail.com
karpagamohan@rediff.com, ppugazhj@rediffmail.com

where LP_k is given by

$$LP_k : \text{maximize } A_kX$$
$$\text{Subject to } A_iX \le b_i,\ i = 1, 2, 3,\ldots, m,\ i \ne k$$
$$X \ge 0.$$

Ilya Ioslovich [10] proposed an approach to find the redundant constraints in the system of equation (1) by using all constraint. The given problem have to optimized 2m times. Then identified the irrelevant constraints.

Hence this approach consumes more number of computational efforts and time. To overcome this difficulty this paper introduces a new method to find the irrelevant constraints. Which is presented in the section 2 and the same section illustrates the new approach with some numerical examples. Section 3 explains the earlier method with numerical examples. The efficiency of the introduced approach is reported through various sizes of small scale LP problems in the section 4. The section 5 draws the conclusion of the paper.

2 PROPOSED METHOD

In this section, a new method is proposed to find the irrelevant constraints. The steps of the proposed method are as follows.

Let us consider the following problem

$$\max Z = \sum_{j=n}^{n} c_i x_i$$

$$\text{subject to } = \sum_{j=n}^{n} a_{ij} x_i \le b_i,\ i = 1,2,3,\ldots,m$$

$$0 \le x_j \le u_j,\ j = 1,2,3,\ldots,n$$

Step:1

Obtain $R_i = \sum_{i=i}^{n} \frac{a_{ij} c_i^T}{a_{ij}^2}$

for each i∈I, I= {1,2,3,...,m}, ($a_{ji} > 0$, $c_j > 0$ $\forall_{i,j}$)

Step:2

Select a most obstructive constraint corresponding to k. Where $k = \arg\min_i (R_i)$,

$1 \le k \le m$

Step:3

Identified the constraints $A_u X \le b_u$, is irrelevant constraint if $\theta_u^k < b_u$ Where θ_u^k is the optimal value of LP_u^k. Where LP_u^k is

LP_u^k: Maximize $\alpha_u^k = A_u X$

Subject to $A_k X \le b_k$

$0 \le X \le U$

The following numerical example illustrates the proposed method and also shows the advantages of the same method by solving LP problems.

Example 1:

Consider the following LPP

Max $z = 4x_1 + 2x_2 + x_3$

Subject to

$2x_1 + x_2 + x_3 \le 30$

$3x_1 + x_2 + x_3 \le 26$

$x_2 + x_3 \le 13$

$x_1 + 2x_2 + x_3 \le 45$

$0 \le x_1 \le 8.67$

$0 \le x_2 \le 13$

$0 \le x_3 \le 13$

Solution:

Here $C^T = (4\ 2\ 1)$

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 3 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 2 & 1 \end{pmatrix}$$

$b^T = (30\ \ 26\ \ 13\ \ 45)$

$U^T = (25\ 20)$

Step 2:

$R_1 = 1.83$

$R_2 = 1.36$

$R_3 = 1.5$

Step 3

$k = \arg\min_i\{R_i\}$, i = 1, 2, 3

k = 2

Solving the problems LP_1^2, LP_3^2 and LP_4^2

LP_1^2 : max $\theta_1^2 = 2x_1 + x_2 + x_3$

Subject to

$3x_1 + x_2 + x_3 \le 26$

$0 \le x_1 \le 8.67$

$0 \le x_2 \le 13$

$0 \le x_3 \le 13$

LP_3^2 : max $\theta_3^2 = x_2 + x_3$

Subject to

$3x_1 + x_2 + x_3 \le 26$

$0 \le x_1 \le 8.67$

$0 \le x_2 \le 13$

$0 \le x_3 \le 13$

and LP_4^2 : max $\theta_4^2 = x_1 + 2x_2 + x_3 \le 45$

Subject to

$3x_1 + x_2 + x_3 \le 26$

$0 \le x_1 \le 8.67$

$0 \le x_2 \le 13$

$0 \le x_3 \le 13$

We have, $\theta_1^2 = 24$

$\theta_3^2 = 20$

$\theta_4^2 = 37$

Since θ_1^2 is less than 30, θ_4^2 is less than 45.
Therefore constraints 1 , 4 are redundant.

Example 2:

Consider the following LPP

Max $z = 20x_1 + 10x_2 + 40x_3 + 20x_4 + 15x_5$

Subject to

$x_1 + 5x_2 + 4x_3 + 2x_4 + 4x_5 \le 120$

$2x_1 + 5x_2 + 2x_3 + x_4 + 0x_5 \le 80$

$4x_1 + x_2 + 5x_3 + 0x_4 + 4x_5 \le 240$

$x_1 + x_2 + x_3 + x_4 + x_5 \le 250$

$0 \le x_1 \le 4.285$

$0 \le x_2 \le 5$

$0 \le x_3 \le 7$

Solution:

Here $c^T = (20\ \ 10\ \ 40\ \ 20\ \ 15)$

$$A = \begin{pmatrix} 1 & 5 & 4 & 2 & 4 \\ 2 & 5 & 2 & 1 & 0 \\ 4 & 1 & 5 & 0 & 4 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$b^T = (120\ \ 80\ \ 240\ \ 250)$

$U^T = (4.285\ 5\ 7)$

Step 2:

$R_1 = 4.173$

$R_2 = 5.58$

$R_3 = 6.03$

$R_4 = 21$

Step 3

$k = \arg\min_i\{R_i\}$, i = 1, 2, 3

k = 1

Solving the problems LP_2^1, LP_3^1 and LP_4^1.

We have $\theta_2^1 = 200$

$\theta_3^1 = 276$

$\theta_4^1 = 96$

Since $\theta_4^1 < b4$, Therefore constraint 4 is redundant.

3 EARLIER METHOD

This method was proposed by Ioslovich [10] to identify the redundant constraints. This method consumes more number of computational efforts and time.

Method:

Solve the problem for each i = 1,2,...,m

Maximize $\alpha_i^f = \alpha_i' x$

Subject to $c^T x \le Z_l$

$0 \le x \le u$

If $\alpha_i^f < b_i$ then the i^{th} constraint is redundant.

Procedure of the method

Step 1:

To find z_i values, solve the problem

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Maximize $z_i = c^T x$
Subject to $a_i' \le b_i$,

Step 2:
Find $y_{u'A}$ and $y_{u'l}$.

Step 3:
Find Max $z_u = c^T x$,
Subject to $y_u' A x \le y_u' b$
$0 \le x \le u$

Step 4:
$Z_l = \min(z_{il}, z_u)$
where $z_{il} = \min_i z_i$.

Example 1:

Consider the example 1 of section 2.
Z_i values are 60, 44.67, 66, 75
Where $y_{1u} = 2$, $y_{2u} = 1.33$, $y_{3u} = 2$, $y_{4u} = 1$. By step 3
$z_u = 60$. Where $Z_{il} = 44.67$. Then $Z_l = 44.67$
$y_{u'A} = (8.99 \;\; 7.33 \;\; 6.33, \;)$ and $y_{u'l} = (165.58)$
Since α_i^f values are 24.84, 34.75, 20, 37.42.
Constraint 1, 4 only identified as redundant constraint by this method for the above example.

Example 2:

Consider the example 2 of section 2.
Z_i values are 1600, 2050, 2930, 3810.
Where $y_{1u} = 10$, $y_{2u} = 20$, $y_{3u} = 5$, $y_{4u} = 0$. By step 3
$z_u = 1809$. Where $Z_{il} = 1760$. Then $Z_l = 1760$
$y_{u'A} = (70 \;\; 105 \;\; 105 \;\; 40 \;\; 60 \;)$ and $y_{u'l} = (4000)$
Since α_i^f values are 251, 200, 343.75, 103.50.
Constraint 4 only identified as redundant constraint by this method for the above example.

4 Numerical Results

The comparative results of the two methods for identifying the irrelevant constraint are presented in the following table. The table shows the comparison results of small-scale problems. Here the numbers of irrelevant constraints are presented. Both these methods identify the same constraints as irrelevant for most of the LP problems. But in some cases proposed approach find many redundant constraints than the earlier method proposed by Ioslovich. However, the proposed method takes very less computational effort and time compared to the Ioslovich method [10] to find the irrelevant constraints in linear programming problems.

TABLE 1: COMPARISON OF TWO METHODS (Small Scale Problems)

S.NO.	Size of the Problem		No. of Irrelevant Constraints Identify by (Irrelevant Constraint Number)	
	No. of constraints	No.of Variables	No. of Irrelevant Constraints (Proposed)	No. of Irrelevant Constraints (Islovich)
1	3	2	1(3)	1(3)
2	3	2	-	-
3	3	2	1(3)	1(3)
4	4	3	2(3,4)	1(4)
5	4	3	2(1,4)	2(1,4)
6	3	3	1(3)	1(3)
7	3	3	1(2)	1(2)
8	4	5	1(4)	1(4)
9	5	4	1(4)	-
10	7	10	4(2,3,6,7)	1(2)

5 CONCLUSION

From this paper, it can be concluded that presolving techniques are highly successful in reducing the size of the input matrices, before they can be sent to an LP solver.
Further, the time taken by the LP solver to solve the presolved problem is significantly less, than the time taken by it to solve the same unpresolved input problems. In this paper, a new approach is proposed to find the irrelevant constraints and compare with Ioslovich[10] procedure. The proposed method takes less time consumption and minimum number of computational efforts in comparison with the earlier method.

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